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Linking cognitive and reaching trajectories via intermittent movement control

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HIGHLIGHTS

- We recorded arm movements and button press responses to random dot kinematograms.
- We described how to fit a Wiener diffusion model to intermittent arm movements.
- We predicted arm movements assuming intermittent access to the decision process.
- This offers the potential for early access to the decision process.

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ABSTRACT

Theories of decision-making have traditionally been constrained by reaction time data. A limitation of reaction time data, particularly for studying the temporal dynamics of cognitive processing, is that they index only the endpoint of the decision making process. Recently, physical reaching trajectories have been used as proxies for underlying mental trajectories through decision space. We suggest that this approach has been oversimplified: while it is possible for the motor control system to access the current state of the evidence accumulation process, this access is intermittent. Instead, we demonstrate how a model of arm movements that assumes intermittent, not continuous, access to the decision process is sufficient to describe the effects of stimulus quality and viewing time in curved reaching movements.

1. Introduction

A core task for cognitive psychology is to uncover the mental states leading up to overt behaviour. Traditionally, theories about cognitive trajectories have been constrained by data recorded from their end point—the behavioural outcome. A recent series of high-profile publications have proposed that fine-grained and direct information about mental states can be found in the trajectories of reaching movements used to indicate the outcomes of decisions. For example, when participants are asked to choose between faces of different races (Wojnowicz, Ferguson, Dale, & Spivey, 2009), words of different categories (Dale, Kehoe, & Spivey, 2007), or numbers of different magnitudes (Song & Nakayama, 2008a), the trajectories of arm movements towards a response target deflect towards the alternative response target in ways that depend systematically on stimulus properties. Such effects suggest a correspondence between physical and mental trajectories whereby the observed reaching trajectory in a decision task serves as a proxy for the underlying mental trajectory through decision space. This view was originally championed by Spivey, Grosjean, and Knoblich (2005) and is now widely accepted. For example, in their influential review, Song and Nakayama (2009), asserted that "the continuity of reaching movements enables each sample point to be modulated by the real-time progress of . . . internal processes" (p. 360; see also Freeman and Ambady (2009), Schmidt and Seydell (2008) and Song and Nakayama (2008a,b)).

The simplicity of the proposed link between mental and physical trajectories is compelling, and convenient. It promises close-to-continuous information about cognitive processing, which has previously been impossible. However, a simple link between mental and physical trajectories seems unlikely given that, in the motor control literature, many have argued that intermittent control is used in generating movements (Fishbach, Roy, Bastianen,

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Miller, & Houk, 2007; Morasso, Casadio, Mohan, & Zenzeri, 2010; Morasso et al., 2010), although others dispute this point of view (see Desmurget and Grafton (2000) for a review). We do not aim to resolve this debate here. Rather, our goal is to demonstrate that a simple *intermittent* model of arm movements can account for the pattern of data observed in a perceptual decision task, providing a more nuanced way to link mental and physical trajectories that is plausible from a motor control perspective. Our intermittent model has been motivated in part by the suggestion of Van der Wel, Eder, Mitchel, Walsh, and Rosenbaum (2009) who proposed in their reply to Spivey et al. (2005) that the cognitive "trajectory" through decision space influences reaching movements at discrete time points.

Our model rests upon the widely held assumption that reaching movements are composed of discrete submovements, analogous to the way in which speech is composed of phonemes (Berthier, 1996; Flash & Henis, 1991; Flash & Hochner, 2005; Konczak & Dichgans, 1997; Krebs, Aisen, Volpe, & Hogan, 1999). Submovements are, by assumption, discrete and ballistic—their amplitude, direction and duration are all determined prior to their onset. This allows us to establish the state of the evidence accumulation process at the onset of a particular submovement.

We exploit the discrete and ballistic properties of reaching submovements using a method inspired by the way that others have exploited the discrete and ballistic nature of eye saccades. For example, in their seminal work, Gold and Shadlen (2000, 2003) had monkeys indicate which direction a collection of dots were moving by performing an eye saccade in the same direction. On some trials, they stimulated the frontal eye fields to prematurely evoke a saccade with a known angle and amplitude (established when stimulation occurred in the absence of a perceptual stimulus). Critically, the landing spot of the prematurely evoked saccades varied systematically with stimulus quality and viewing time: the longer the monkey viewed the random dot kinematogram prior to being stimulated, the more the landing spot of the evoked eye saccade was deflected in the direction suggested by the stimulus. In this way, Gold and Shadlen were able to map out the time course of evidence accumulation in a simple perceptual decision task.

We designed a procedure inspired by this technique, using premature arm movements. We had human participants indicate their decisions by reaching towards targets, and we "evoked" premature movements by requiring them to start moving just after stimulus onset. Using this procedure, we show that the movements generated are consistent with predictions about partially-completed processing in a standard cognitive decision theory. This establishes that it is possible to link mental and physical trajectories via a plausible intermittent motor control system, rather than the simple direct mapping that is usually assumed.

Our domain of application is simple perceptual decision-making, which has been studied extensively by both cognitive psychologists (Green & Swets, 1966; Luce, 1986; Ratcliff, 1978; Smith & Vickers, 1988; Usher & McClelland, 2001) and, more recently, neuroscientists (Gold & Shadlen, 2000, 2003; Hanks, Ditterich, & Shadlen, 2006; Newsome, Britten, & Movshon, 1989; Reddi & Carpenter, 2000; Roitman & Shadlen, 2002). Mathematical models of these processes (Brown & Heathcote, 2005, 2008; Ratcliff & Rouder, 1998; Ratcliff & Smith, 2004) are generally based on the notion that evidence is accumulated until a bound is reached, at which point a decision is made, and the models have traditionally been evaluated by data collected about the endpoint of the process, the response time. Nevertheless, there have been previous attempts to observe the evidence accumulation process before the final decision has been made. The most popular approach uses imperative signals designed to interrupt the decision process and force a premature response (Meyer, Irwin, Osman, & Kounios, 1988). However, as these techniques necessarily change the task by forcing a response, it is not clear whether they actually allow us to observe the accumulation before a final response is produced. For example, Ratcliff (1988, see also (2006)) showed that response signal techniques were unlikely to be able to address crucial questions, such as the ability of partially-completed processing to inform decision-making. Our work establishes that reaching movements might help solve this problem. Rather than requiring interrupted cognitive processing in order to probe the evidence accumulation process, our method naturally generates two types of movements. Some movements are straight to a target; these occur when a decision has been made before movement onset, and are analogous to standard button press responses. Other movements begin before a final decision has been made. The subjects spontaneously produce both types of trials, making it plausible that the same evidence accumulation process is occurring in both cases.

We test whether an intermittent model of arm movements in a perceptual decision making task can describe the data. Our perceptual decision task uses random dot kinematograms, with some decisions indicated by reaching movements to targets on a touch screen, and others by standard button presses. By analysing the directions of premature movements, and also by fitting models to both the movement data and button press reaction time (RT) data, we demonstrate that a model based on intermittent arm movements is sufficient to describe the observed reaching data, and that these findings are compatible with a standard RT model.

2. Material and methods

2.1. Participants

Three right-handed men participated in these experiments. All three were healthy with no known neurological or peripheral disorders, and had normal or corrected-to-normal vision. All gave informed consent according to the policies of the Macquarie University Human Research Ethics Committee.

2.2. Stimuli

The stimuli were presented on an LCD touch screen (70 cm $\times$ 39 cm, 1360 $\times$ 768 pixels, 60 Hz). The stimuli used in this experiment were random-dot kinematograms (Gold & Shadlen, 2000), contained within a circular 5° aperture at the centre of the screen. An average of 7 dots were shown in each frame. Each dot was a white square with side lengths of 2 pixels (equivalent to a viewing angle of 0.087° per side). At each frame, the probability that the dot moved in a given direction (at 5°/s) rather than being redrawn at a random position was determined by the coherence (at one of 5 levels: 3%, 6%, 12%, 24%, 48%). The stimuli were shown for 300 ms. The direction of motion (left or right) was randomly selected between trials. The stimuli were generated using the Psychophysics toolkit (Brainard, 1997; Pelli, 1997) and custom Matlab code. The subjects indicated the direction of the movement of the dots by either reaching to touch a target on a touchscreen or pressing a button. The same stimuli were used with both types of response.

2.3. Procedure

For the arm reaching experiments, the subjects were required to point with their right index finger to a target on the touch screen corresponding to the direction of movement of the random dots. The targets were 5 cm $\times$ 5 cm boxes, shown on the left and right sides of the screen, centred vertically. For these tasks, the subjects began with their index finger resting on a button, and were required to lift their finger and start moving towards the screen within 350 ms after stimulus onset (but not before onset). This requirement for premature initial movement was designed to occasionally produce initial movements that are not directed to one target or another, motivated by the prematurely evoked saccades from Gold and Shadlen's (2000, 2003) design. Participants were required to move forward continuously (defined as the position of the finger being closer to the screen in each frame compared to the previous frame more than 80% of the time) until they touched the screen. If they did not start within 350 ms, or move forward continuously, the trial was aborted and feedback presented (a loud buzz and an error message). These trials (less than 5%) were not included in the analysis.

Subjects were required to look at a fixation point in the centre of the screen before presentation of each stimulus. Each trial began by pressing a pedal with the right foot. Two seconds after pressing the pedal, the stimulus was shown. Feedback was given to the subjects by showing text on the screen, indicating whether they selected the correct target.

Five days of testing were performed for each subject. On each day of testing, 500 trials were recorded, divided into 10 blocks. Each block consisted of five repetitions of the five coherence levels in the two directions. After a first day of practise, two additional sessions were recorded. Two more sessions with the same stimuli but with button pressing rather than arm movements were recorded on alternate days, counterbalanced across subjects. On each day, subjects performed either the button press task or the reaching task. The first day for all subjects consisted of arm reaching; the data from this day were not used in further analysis.

For the button press experiments, the subjects were required to press a button corresponding to the direction of movement of the random dots, while in the reaching tasks they had to reach and press a square on the screen in the direction of movement of the dots. In both tasks, the subjects were instructed to respond in a fast and accurate way.

2.4. Data acquisition

For the button-press experiments, the reaction times were recorded with a button box connected to a PCI-DIO24 data acquisition card (Measurement computing, Norton, MA, USA), with a sampling rate of 6000 Hz. The arm pointing movements were recorded using an Optotrak Certus (Northern Digital, Ontario, Canada), which sampled at 200 Hz the 3D position of two infrared markers placed on the index finger tip.

2.5. Data analysis

2.5.1. Reaction time (RT) data

We used a standard Wiener diffusion process to model the evidence accumulation process leading up to making a decision. The Wiener diffusion model is a sequential sampling model—subjects repeatedly sample information about the stimulus, which can be transformed into a single dimension and added to the previous information in an accumulator. When the accumulator reaches an upper or lower bound, the appropriate decision is made. The accumulation is a noisy process, with the mean rate of information accumulation towards the upper boundary given by the *drift rate*. The other parameters describing the process are the standard deviation of the change in a given time step, the starting value of the accumulation (the *bias*), and the upper boundary, with the position of the lower boundary arbitrarily set to zero. The predicted reaction time is given by the sum of the time taken for the Wiener diffusion process to reach a bound added to the *non-decision time*, which accounts for the time taken to encode the stimulus and make the motor response.

The parameters of the best-fit Wiener diffusion model were calculated from the RT data, using maximum likelihood estimation (Myung, 2003). The probability density function (PDF) of the Wiener first passage times was estimated using the algorithm described by Navarro and Fuss (2009), with parameter search by a combination of the simplex method followed by simulated annealing (Kirkpatrick, Gelatt, & Vecchi, 1983; Vandekerckhove, 2006). The Wiener diffusion model has four free parameters—the drift rate v, the bias z_0, the upper boundary a, and the nondecision time T_{er} (which we considered to be constant across trials for each subject). The standard deviation s was fixed at 1. A separate drift rate was fit for each condition, the other parameters were assumed to be identical across conditions.

2.5.2. Movement data

The arm trajectory data were first decomposed into submovements. Each submovement was modelled according to the minimum jerk criterion (Flash & Hogan, 1985), which specifies the velocity profile $\dot{x}(t)$ as

$$\dot{x}(t) = \frac{A_x}{D}\left(30\left(\frac{t-T_0}{D}\right)^4 - 60\left(\frac{t-T_0}{D}\right)^3 + 30\left(\frac{t-T_0}{D}\right)^2\right) \quad (1)$$

where T_0 is the starting time, D is the duration and A_x the amplitude. According to this equation, each submovement follows a straight-line trajectory, although the superposition of multiple submovements can lead to curved movements (Flash & Henis, 1991). The velocity profile of the reconstructed movement $F(t)$ consists of the summation (or superposition) of N overlapping submovements:

$$F(t) = \sum_{i=1}^{N} \begin{cases} 0 & t < T_{0i} \\ \dot{x}(t) & T_{0i} \le t \le T_{0i} + D \\ 0 & t > T_{0i} + D. \end{cases} \quad (2)$$

Each submovement was considered to be 2-dimensional (in the two horizontal directions), vertical movements were not considered due to the relatively small amount of movement in this direction. Each submovement is described by four parameters: the starting time T_0, the duration D, and two parameters describing the amplitude, A_x and A_y.

In order to measure the quality of the reconstruction, an error measure was defined

$$E = \sum_t \frac{(F_x(t) - G_x(t))^2 + \left(F_y(t) - G_y(t)\right)^2 + \left(F_v(t) - \sqrt{G_x(t)^2 + G_y(t)^2}\right)^2}{2\left(G_x(t)^2 + G_y(t)^2\right)} \quad (3)$$

where G_x and G_y are the x and y components of the measured trajectory, and F_v is the reconstructed tangential velocity. The tangential velocity term is necessary to prevent the optimization procedure from selecting approximately simultaneous submovements in opposite directions, which are implausible.

The best reconstruction of the movement was found using the constrained nonlinear optimization function in the Optimization toolkit of Matlab (with the Trust-Region-Reflective algorithm), with constraints on the parameters of

$$\begin{aligned} &0 \le T_0 \le T_f - 0.167 \quad (4) \\ &0.167 \le D \le 1.0 \\ &-5 \le A_x \le 5 \\ &0.1 \le A_y \le 5 \end{aligned}$$

where T_f is the time at the end of the recorded movement. The first two constraints require submovements to have a duration of at least 167 ms as in Rohrer and Hogan (2006). The third constraint

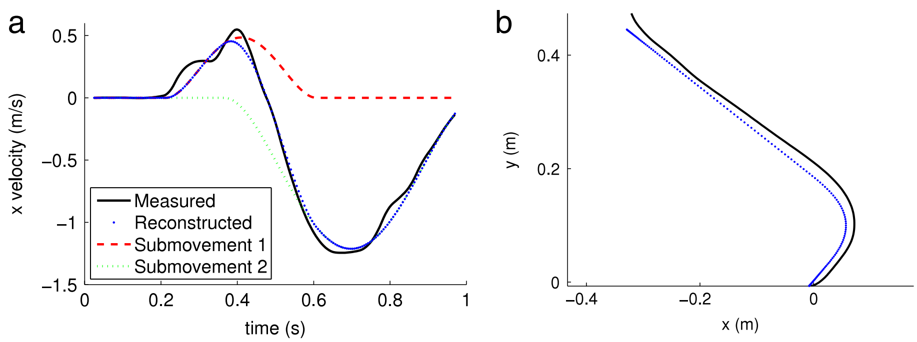


Fig. 1. Sample decomposition. In this trial, the subject began moving towards the right at approximately 200 ms, then corrected with a submovement to the left at approximately 400 ms. (a) Shows the velocity in the x (left–right) direction, and (b) shows the position (x is left–right, y is forward-back).

simply reflects the physical limits of the reachable area. The fourth constraint, requiring the amplitude to be positive, reflects the procedural requirement to constantly move forward during the movement.

The initial "guesses" were randomly chosen within the above constraints, and the procedure was repeated 10 times to increase the probability of selecting the globally optimal solution (Rohrer & Hogan, 2006). The above procedure was repeated for 1–4 submovements. The number of submovements to use for a particular trial was selected as the smallest number of submovements with an error measure of less than 0.03. When four submovements were unable to produce an error of less than 0.03, this trial was not used in further analysis (<1% of the trials). The threshold of 0.03 was selected based on pilot studies to allow an accurate representation of the movement without producing spurious submovements. An example of submovement decomposition is shown in Fig. 1.

From these decompositions, the onset time of the submovements, and their amplitude (in the x/y directions) were extracted. Overwhelmingly, 1, 2 or 3 submovements were used (>98% of the movements for all subjects). The movements reconstructed from the submovements (i.e., using Eq. (2)) accurately captured the movement, with an average reconstruction error (Eq. (3)) of 0.0160 $\pm$0.0018 (averaged across subjects).

As a model-free test of our predictions (i.e., without assuming a decision model), we compared the initial heading angles of the trajectory, separately for different types of stimuli. This was calculated as the slope of the trajectory in the horizontal plane after moving 4 cm towards to the screen. We use the slope of the trajectory rather than the angle of the line from the starting point to this point in order to minimize the effects of very small initial movements related to lifting off the button.

2.5.3. A Wiener diffusion model for movement data: overview

We modelled the movement data in the simplest way we could, while still respecting the constraints imposed by intermittent motor control. Our model makes the key assumption that the decision process is identical between the arm movement and button press halves of the experiment. In both cases, we use the simple Wiener diffusion process described above. To model the arm movement data, we assume that a motor control process operates in parallel, which initiates reaching movements at various points during the trial. The motor control process is constrained to respect the demands of our experimental procedure: participants were required to initiate movements quickly after stimulus onset, and to keep moving continuously forward during a trial. The motor control process models these demands by beginning with a process that triggers an initial movement soon after stimulus onset, to ensure that the movement initiation deadline is met. Algorithmically, we use a standard process for simple detection—a one-sided diffusion process, although we also allow that movements can be initiated more quickly in the case that the decision process terminates before the movement initiation process. Next, we assume that new submovements are initiated whenever an existing submovement is about halfway through (we estimate the precise proportion from the data). This constraint reflects the requirement that participants more forward continuously.

Each time the movement generating process initiates a new submovement, the direction of that movement is determined by querying the current state of the decision process. If the decision process has finished, the submovement is one that reaches all the way to the target. If the decision process has not finished, the submovement is one that moves *partially* towards whichever target is most likely given the current state of the decision process.

2.5.4. Fitting the Wiener diffusion model to arm movement data

Initial movement times for the movement generation process are modelled as a race between the movement initiation process (i.e., triggered by the stimulus onset) and the Wiener diffusion process for the decision task. As in other examples of onset detection (e.g., Heathcote (2004)), we model the movement initiation process using a one-sided diffusion process, which results in a Wald distribution for finishing times.

According to our model, arm movements that reach the target using a single submovement correspond to trials on which the Wiener decision process had already terminated before the movement initiation process (from the Wald process) had terminated. The associated likelihood function is given by the probability that the Wiener diffusion process has reached the appropriate bound, i.e., the probability density (PDF) of the first passage time, multiplied by one minus the cumulative distribution (CDF) of the Wald distribution for the movement initiation process:

$$\begin{aligned} &L_{\text{OneSubmovement}}\,(t_0, r)\\ &\quad= P_{\text{Wiener}}\,(x_1 = a_1 r,\, T = t_0;\, z_1,\, v_1,\, a_1,\, T_{er1},\, s)\\ &\qquad\times \left(1 - P_{\text{Wald}}\left(x_m = a_m,\, T \le t_0;\, v_m,\, a_m,\, T_{er,m},\, s\right)\right) \end{aligned} \tag{5}$$

where t_0 is the movement onset time, r is a binary variable indicating the final response (either 0 or 1, corresponding to a decision to move left or right respectively), $P_{\text{Wiener}}(x_1 = a_1 r, T = t_0)$ is the PDF of the Wiener first passage time (Navarro & Fuss, 2009) and $P_{\text{Wald}}(x_m = a_m, T \le t_0)$ is the CDF of the Wald distribution (Heathcote, 2004). The parameters of the Wiener diffusion process for the decision are the relative starting point (z_1), drift rate (v_1), bound (a_1) and the non-decision time (T_{er1}). Only the drift rate was allowed to vary by condition. For the movement initiation process, the parameters are the drift rate (v_m), the bound (a_m) and the non-decision time ($T_{er,m}$). For the movement initiation process, the same parameters were used for all conditions, as we assume that this process is driven by the appearance of the stimulus and not the quality of the stimulus. For both processes, the standard deviation of the drift rate within trials (s) was fixed arbitrarily at 1.0.

A trial with two or more submovements is predicted when the movement initiation process generates a movement before the decision process has completed. In these cases, we have previously shown that subjects move towards intermediate targets with their first submovement (Friedman & Finkbeiner, 2011). We assume that the direction of these first submovements (to the left or to the right) is guided by the current state of evidence accumulation. The likelihood for these trials, conditioned on the direction of movement, is given by multiplying three probabilities: (1) the probability that the decision process has not yet finished before movement onset (t_0) and the direction of the initial movement is linearly related to the current value of evidence accumulation; (2) the probability that the movement initiation process initiates a movement at time t_0; and (3) the probability that the decision process has terminated before the onset time of the second submovement (t_1), but after the onset of the first movement (t_0). An initial submovement in the direction θ_1 followed by a second submovement (to the left if $r = 0$, or to the right if $r = 1$) would have the following likelihood:

$$\begin{aligned} &L_{\text{TwoSubmovementsWithPartial}}(t_0, t_1, \theta_1, r) \\ &\quad = P_{\text{Unterminated}}\left(x_1 = \frac{\theta_1 - \theta_l}{\theta_r - \theta_l} a_1, T = t_0; z_1, v_1, a_1, T_{er1}, s\right) \\ &\qquad \times P_{\text{Wald}}\left(x_m = a_m, T = t_0; v_m, a_m, T_{er,m}, s\right) \\ &\qquad \times P_{\text{Wiener}}\left(x_1 = a_1 r, t_0 < T < t_1; z_1, v_1, a_1, T_{er1}, s\right) \end{aligned} \tag{6}$$

where $P_{\text{Unterminated}}(x, t)$ is the PDF of the non-terminated Wiener process (Eq. (1) in Ratcliff (1988)), i.e. the probability that the Wiener process has not reached either bound before time t and the current state of accumulation is x. θ_l and θ_r are the directions of the left and right targets respectively. We note that the first term in Eq. (6) linearly transforms the initial direction θ_1 into the amount of evidence accumulated, such that an initial movement in the direction of the left target (θ_l) corresponds to $x_1 = 0$ and a movement in the direction of the right target (θ_r) corresponds to $x_1 = a_1$.

The second term in Eq. (6) is given by the PDF of the Wald process. The third term is computed by integrating the PDF of the Wiener first passage time between t_0 and t_1. We integrate over the time between t_0 and t_1 rather than computing the PDF of the Wiener first passage time at t_1 because we assume that the second submovement begins after a fixed proportion of the previous submovement has been executed and not at the time when the bound is reached. The second submovement generally begins when the first submovement is around halfway through (Lee, Port, & Georgopoulos, 1997), but we estimated the exact proportion from the data.

If the decision bound has not been reached by the time the second submovement is initiated, then the second submovement will be a *partial* movement and a third submovement will be required to take the hand all the way to the target. As these three submovement cases were very rare in our data, we use the same likelihood expression (Eq. (6)), where t_1 *is* the onset time of the last submovement.

In order to test the claim that partial accumulation is used in generating the reaching movements, we also tested a version of the model that does not use partial information. This follows the proposal of Van der Wel et al. (2009), that an initial movement is made in a direction between the targets regardless of the state of evidence accumulation at that point in time, and later an additional submovement is made towards the selected target. For one-submovement cases, the likelihood expression is equivalent (Eq. (5)). When a final decision has not been made before the movement initiation process reaches a bound (i.e., at least two submovements need to be made), we assume in this case that the initial submovement is uninformed by the partial evidence accumulated, so that the likelihood function is given by the joint probability that the Wiener process has not completed, and the Wald process has completed:

$$\begin{aligned} &L_{\text{TwoSubmovementsWithoutPartial}}(t_0, t_1, r) \\ &\quad = P_{\text{Wiener}}\left(x_1 = a_1 r, t_0 < T < t_1; z_1, v_1, a_1, T_{er1}, s\right) \\ &\qquad \times P_{\text{Wald}}\left(x_m = a_m, T = t_0; v_m, a_m, T_{er,m}, s\right) \end{aligned} \tag{7}$$

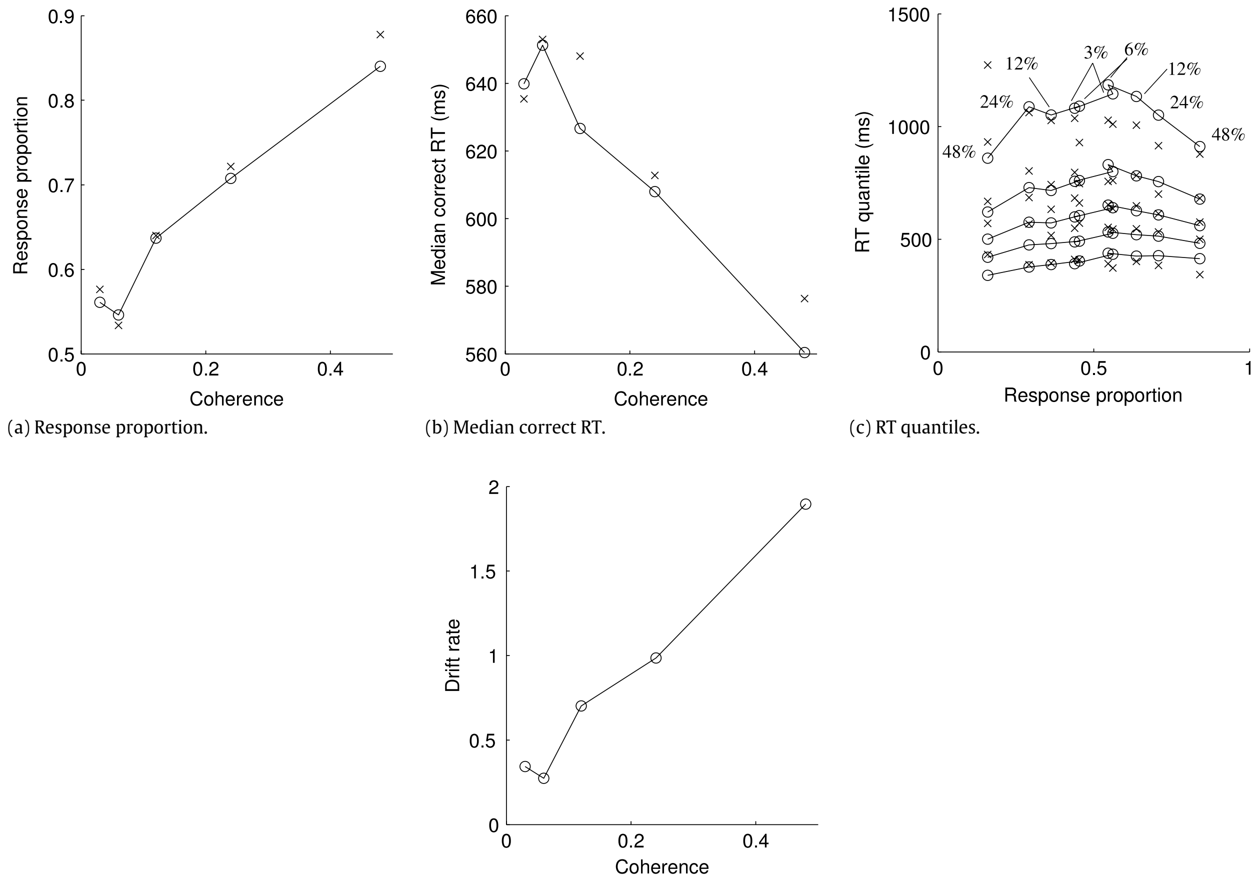
(where P_{Wiener} indicates the CDF of the decision process and P_{Wald} indicates the PDF of the movement initiation process).

We used maximum likelihood estimation to find the optimal parameters describing the two processes for the two models. As with the RT data, we used the simplex method, followed by simulated annealing to find the parameter values that maximize the likelihood.

In order to compare the predictions of the models with the experimental data, for the reaction time data, we estimated the response proportion, the median reaction time, and the reaction time quantiles (for correct and incorrect trials), by simulating the Wiener diffusion process 10,000 times for each coherence level, using a random walk approximation (Tuerlinckx, Maris, Ratcliff, & De Boeck, 2001). For the reaching data, we simulated both the decision process (Wiener diffusion) and the movement generation process 10,000 times for each coherence level, for both models (with and without partial information). In addition to comparing the accuracy of the predictions, we also compared the movement onset times, the proportion of trials that complete using just a single submovement and the distribution of maximum "path offsets". Path offset is defined as the maximum perpendicular distance of the trajectory from a straight line leading from the initial point to the target (Finkbeiner & Friedman, 2011). The distributions of predicted maximum path offset were compared to the data using a likelihood ratio test. The probability of the path offset being in 15 equally spaced bins from 0 to 0.5 was calculated from the 10,000 model simulations, for the two models. The likelihood of the observed data count in each bin was then calculated, using Snodgrass and Corwin's (1988) correction, i.e. setting the probability to $0.5/N$ when the observed or model count was zero. The two models were then compared via a likelihood ratio test, for each subject.

To simulate the movement generation process, we had to make several assumptions about how movements are generated. For one submovement trials (when the bound was reached before movement onset), the amplitude and angle are fixed by the target location. The duration of the movement was sampled from a normal distribution, fit to the durations of one submovement trials for that subject. For two submovement trials (when the bound had not been reached at movement onset), the amplitude of the first submovement was sampled from a normal distribution. As the duration and amplitude are correlated (larger movements take longer), the durations (for the first and second submovements) were calculated as a function of the amplitude of the appropriate submovement, again using the relationship fit from the data for that subject. For the model using partial information, based on Eq. (6), we assume that the angle on the ith trial of the first submovement is:

$$\theta_i = (\theta_r - \theta_l) \frac{x_{1i}(t_0)}{a_1} + \theta_l \tag{8}$$



(a) Response proportion.

(b) Median correct RT.

(c) RT quantiles.

(d) Drift rate.

Fig. 2. (a) Response proportion, (b) median correct RT, (c) RT quantiles and (d) drift rates as a function of coherence, for the button press experiments. In (a–c), the solid line and the *o*'s are the model prediction, while the *x*'s are the data, all averaged across subjects. In (c), the 0.1, 0.3, 0.5 (median), 0.7 and 0.9 RT quantiles are plotted against response proportion, with correct responses plotted towards the right, and error responses plotted towards the left.

where $x_{1i}(t_0)$ is the state of the Wiener decision process at movement onset (generated from a random walk), a_1 is the bound of the Wiener process. Eq. (8) captures the idea that the angle of movement should match the amount of accumulated information. For the model that does not use partial information, we assume that the initial angle is straight ahead ($\theta = \pi/2$).

The second submovement was assumed to start a certain proportion of the way through the first submovement (usually around half); the exact value was fit using regression from the data. In these two submovement trials, the target is selected based on which bound was reached in the meantime, specifying the amplitude and angle of the second submovement. The duration of the second submovement is similarly sampled from a normal distribution, fit from the data. The submovements were added using Eq. (2) to produce the predicted trajectory for a single trial. Due to the relatively low probability of three submovement trials, in the simulation we only considered one or two submovement movements. The mean parameters used for predicting the trajectories are given in the Appendix.

3. Results

3.1. Reaction time data

Fig. 2 shows the data and predictions from the model for the reaction time data, from the button-press condition only. Decision accuracy and response speed (Fig. 2(a) and (b)) increased with coherence, as expected, and the model predictions closely match the data. Fig. 2(c) compares the reaction time quantiles (0.1, 0.3, 0.5, 0.7, and 0.9) between the model (circles) and the data (crosses). While there are some discrepancies between the predictions and the data, the quantile plots illustrate that the model did not merely predict mean reaction time correctly, but also predicted the full distribution of reaction times, for both correct and incorrect responses. The estimated parameters for the model are in the first row of Table 1.

3.2. Movement data

The predicted finishing time distributions for the movement generation process (for the model using partial information), together with the actual movement onset times are shown in Fig. 3, for data from one subject. The top row shows the probability density of finishing times of the movement initiation process (line with circles) and the decision process (dot–dash line). The movement initiation time predicted by the model (dashed line) is based on whichever of these processes terminates first. It closely matches the onset times for the data (solid line). The second row breaks down these onset times into the single-submovement and more-than-one-submovement cases. Reaching trajectories that commence very early after stimulus onset always have more than one submovement because the decision process has not had enough time to reach a bound. At later times, there is a mix of one and two submovement trials depending on whether the decision process terminates before the movement initiation process. The relative proportion of one and two submovement trials changes as a function of time, and is also modulated by the rate of evidence accumulation (determined by the coherence).

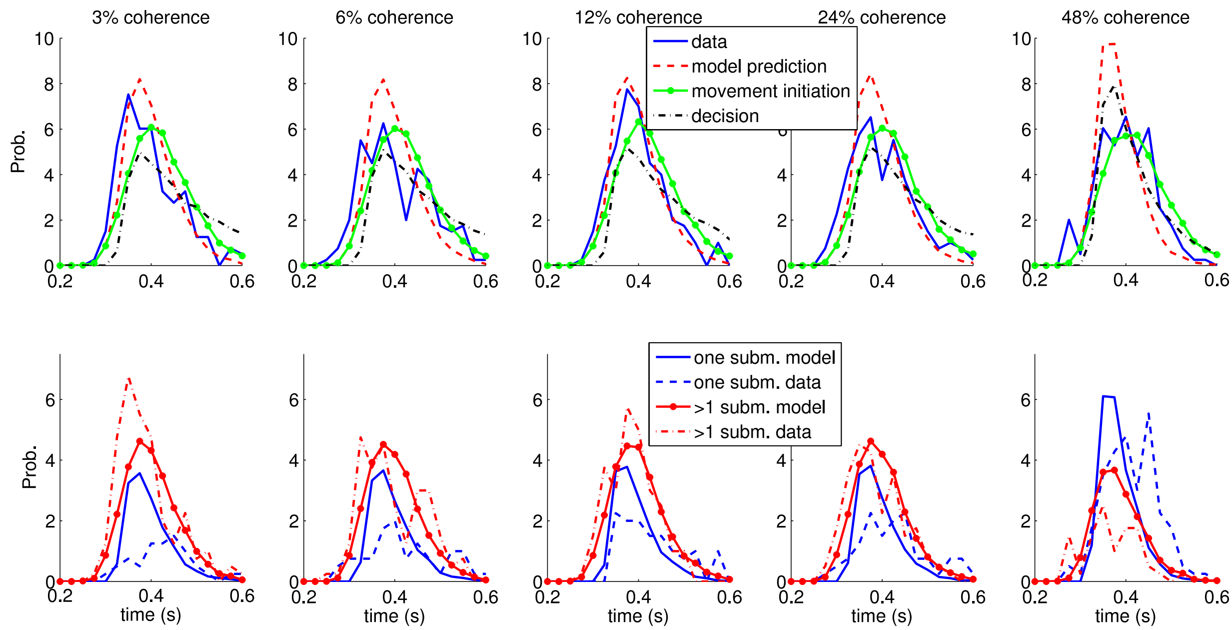
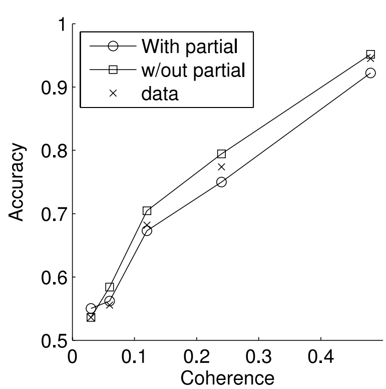
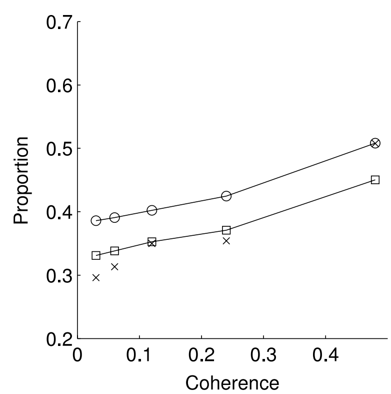


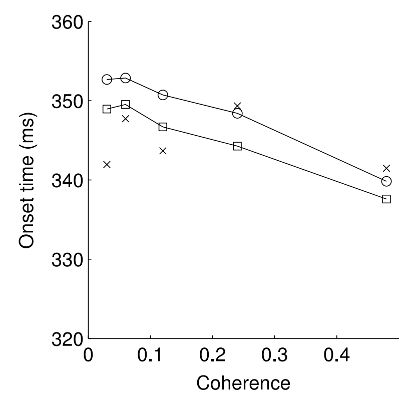
Fig. 3. The top row shows the probability density of when the movement initiation process (line with circles) and the decision process (dot–dash line) reach their bounds. Whichever of these processes reach the bound first cause the movement to be initiated (dashed line). These model predictions are similar to the data (solid line). The second row shows the probability density of observing one or two submovement movements as a function of movement onset time. When the decision process reached the bound first, one submovement was produced; otherwise, more than one submovement was produced. The model predictions (solid line and line with circles) are similar to the observed data (dashed and dot–dash lines). All data is shown for a representative subject for the model that uses partial information.



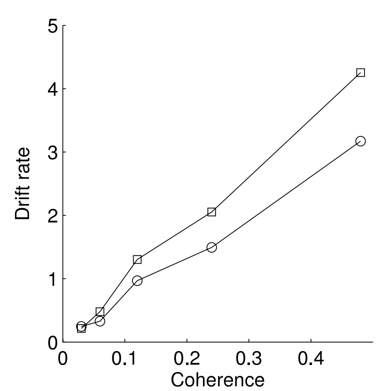
(a) Response proportion.



(b) Proportion of one submovement trials.



(c) Movement onset time.



(d) Drift rate.

Fig. 4. (a) Response proportion, (b) proportion of one submovement trials, (c) movement onset time and (d) drift rate, as a function of coherence, for the arm movement experiments for both models.

From these predictions for both models and the data we computed the accuracy, proportion of single submovement trials, and the movement onset time, and compared the values, shown in Fig. 4. We note that while both models fit the same 11 parameters, due to the differences in likelihood functions, the two models produce different estimates of these parameters, leading to different predictions of accuracy and the other measures. The accuracy is calculated in the same way as for the reaction time

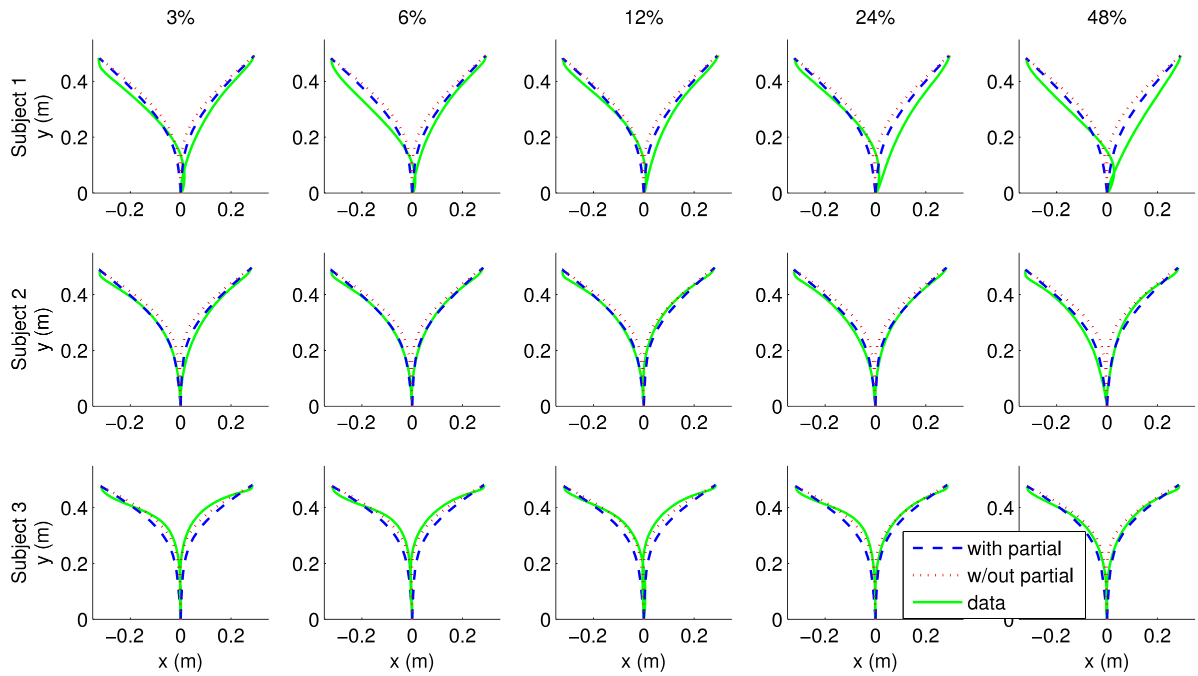


Fig. 5. Mean performed trajectories (solid lines) and mean predicted trajectories for the model with partial information (dashed lines) and the model without partial information (dotted lines), for two-submovement movements for the 5 coherence levels, by subject.

Table 1
Mean parameter fits to the Wiener diffusion model for the reaction time and arm movement experiments. The experiments were performed by the same subjects, on different days. The parameters are the bound (a_1), relative starting point (z_1), mean drift rates for condition i (v_{1i}) and the non-decision time (T_{er1}). The standard deviation (s) was fixed at 1.0. For the arm movements, there were three additional parameters describing the deadline process: the bound (a_m), the drift rate (v_m) and the non-decision time ($T_{er,m}$).

Experiment	Reaction time	Arm movements (with partial)	Arm movements (without partial)
a_1	1.28	0.85	0.75
z_1	0.6	0.46	0.39
v_{11}	0.34	0.31	0.23
v_{12}	0.27	0.4	0.59
v_{13}	0.7	1.34	1.59
v_{14}	0.99	1.96	2.42
v_{15}	1.9	3.96	4.72
T_{er1} (s)	0.32	0.28	0.29
a_m		2.08	2.38
v_m		9.47	9.11
$T_{er,m}$ (s)		0.13	0.08

experiment (based on the first time a bound is reached for the decision process). The models fits the data well, but have a tendency to predict too slow movement onset times in the hardest conditions (low coherence). The models predict a small (∼15–20 ms.) decrease in movement onset times with increasing coherence. This is because as the coherence (and drift rates) increase, the likelihood that the decision process finishes before the movement initiation process increases. The small size of this predicted effect means that it could not be detected in data without much larger samples.

Predictions for the mean trajectory for trials with more than one submovement in the different conditions are shown in Fig. 5, for each of the three subjects. The model's predictions match the data quite well, appropriately capturing changes in curvature with coherence. Fig. 6 shows the differences in mean trajectory between the extreme coherence levels (3% and 48%) for the data and the two versions of the models, for trials with more than one submovement. We note that the model where partial information is used shows trajectories that depend on coherence, whereas the model which does not use partial information does not show differences between the coherence levels. It should be noted that these predictions are dependent on the assumption that the onset time of the second submovement is independent of the state of evidence accumulation.

In order to quantitatively compare the trajectories, we computed the maximum path offset for each trial. The distributions of the path offset are shown in Fig. 7 for each of the subjects. The bar chart is a histogram of the maximum path offset calculated from the experimental data, the dotted line is for the model that does not use partial information, and the dashed line is for the model that does use partial information. We note that the movements generated by the model that does not use partial information have a larger maximum path offset (i.e., are more curved) than movements generated by the model that does use partial information. This is because the first submovements are uninformative (i.e. move straight ahead), and so require a larger corrective movement, causing the entire movement to be, on average, more curved. We compared the distributions using a likelihood ratio test, where we found that the model with partial information predicted the distribution of data much better than the model without partial information (the difference in log-likelihood was greater than 39 for all subjects). The log-likelihoods for the subjects were 39.9, 146.6 and 205.9, (positive values indicate the model with partial information is more likely than the model without partial information).

An alternative way to investigate whether partial information plays a role in movement is to examine the initial direction of movements on only those trials in which movement began before a decision was reached. The intermittent account of motor control in terms of submovement makes this simple: trials in which movement begins before a decision is complete are just those trials with more than one submovement. Following this, the account that assumes availability of partial evidence predicts that, on trials with two-submovements, the nature of the stimulus should influence the direction of initial movement; the account that assumes no availability of partial evidence assumes that the direction of initial movement should be independent of the stimulus. To test these predictions, we compared the initial heading angles of two-submovement movements between trials using left- vs.

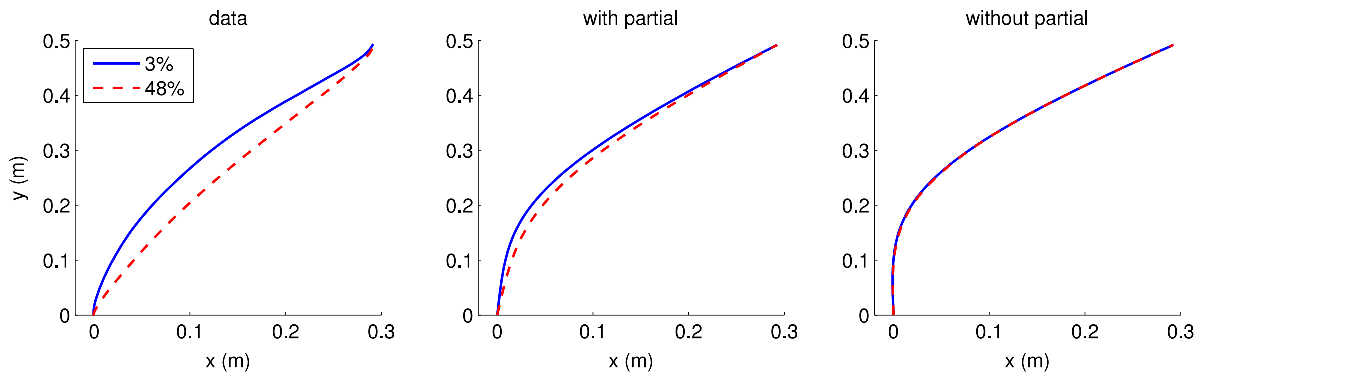


Fig. 6. Comparison of the mean trajectories between the 3% coherence and the 48% coherence for subject 1. The first panel shows the data, the second the predictions of the model using partial information, and the third panel shows the predictions of the model without using partial information.

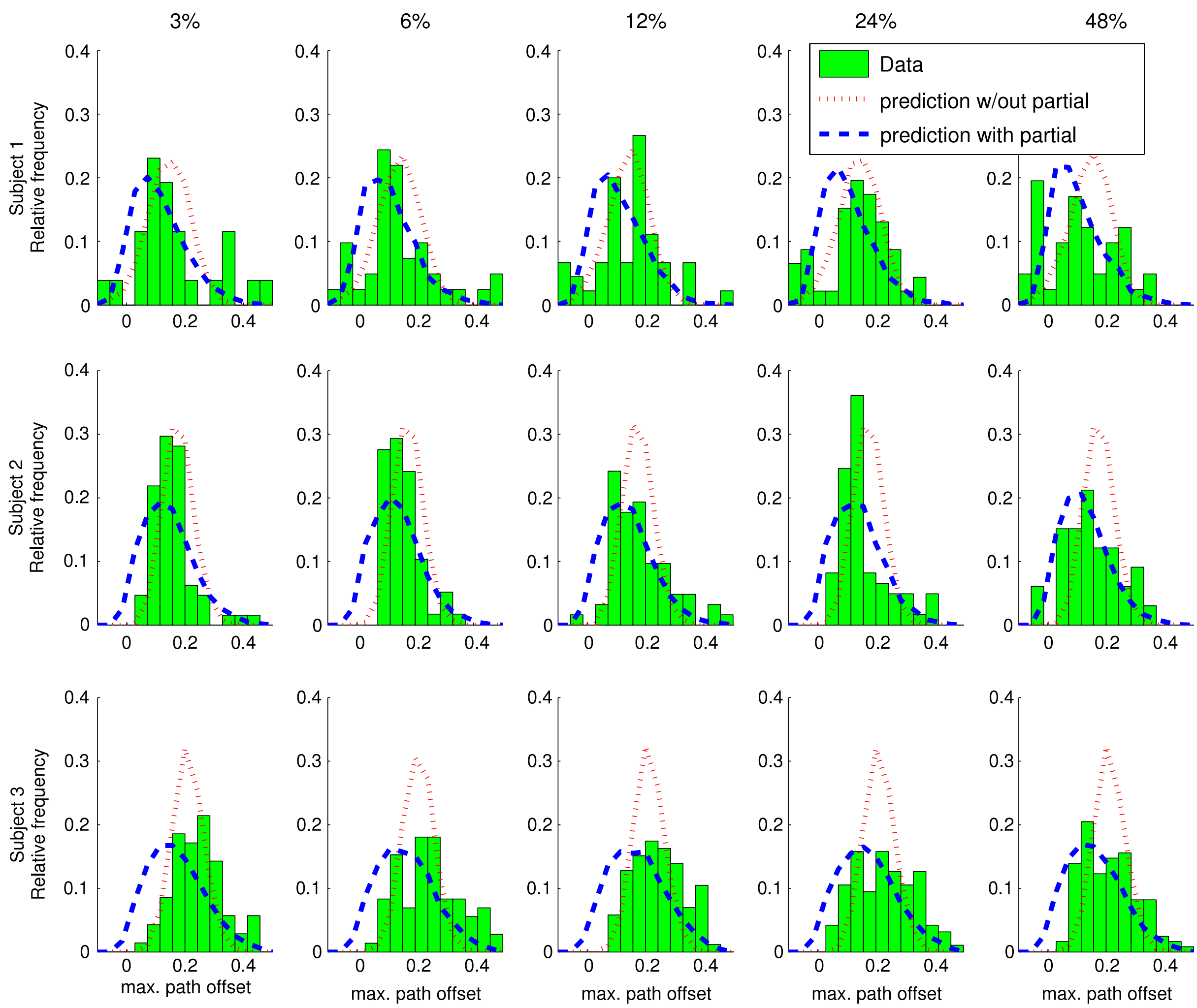


Fig. 7. Histograms of path offset for the data (bars), and the model predictions (with partial information is the dotted line, without partial information is the dashed line). Each row is a different subject, and each column a different coherence level.

right-favouring targets. There was a significant difference ($t-$test, $t(6838) = 3.36, p < 0.001$, effect size (Cohen's d) $= 4.66°$) between movements to the left target (mean angle 88.7°) and movements to the right target (mean angle 87.7°).

3.3. Comparison of response methods

The accuracy of the subjects in the two experimental methods (button press [RT] and reaching movements) is compared in Fig. 8. For most of the subjects and conditions, the accuracy was slightly higher in the reaching conditions. However, in the reaching conditions, the entire movement took longer and allowed the opportunity to correct while the hand was mid-flight. The differences in accuracy were relatively small between the two response types. In addition, while the numerical values of the drift rates were different, for both the button press and reaching models an approximately linear increase with coherence was observed (see Fig. 2(d) and Fig. 4(d)). These similarities support a key

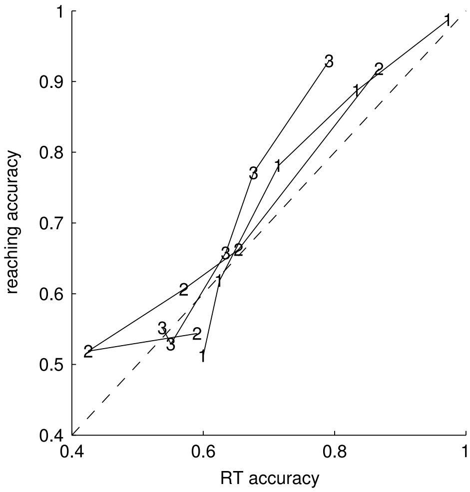


Fig. 8. Comparison of the accuracy between the two experiments (RT and reaching). The dotted line indicates equal performance/parameter values between the two experimental conditions. The numbers indicate the accuracy values for the corresponding subject.

assumption of our modelling: that a common decision process subserves the reaching and button press decisions.

4. Discussion

Our purpose was to determine whether an intermittent model of arm movements was a plausible way to link movement trajectories and cognitive processing, using decision making with a random dot kinematogram task. Our model rests upon the assumption that reaching movements are composed of discrete submovements. We assumed that a separate process to the evidence accumulation process generates these submovements at one or more times during a decision. We found that a model that generates intermittent movements based on the current state of evidence accumulation (i.e., whether a decision bound has been reached at movement onset) was sufficient to describe the main features of these movements.

The modelling techniques described here showed that arm movements can be linked to underlying decision processes. The link we have drawn, using submovements, is similar to the kinds of links previously drawn using reaction times from deadline-based choice experiments (e.g., Ratcliff (1988, 2006)), and using evoked eye movements in monkeys (e.g., Gold and Shadlen (2000, 2003)). Our way of linking arm movements and decision processing is quite different, however, from the direct mapping often assumed (e.g., Song and Nakayama (2009)). Rather, we have shown that it is sufficient to assume that the current state of evidence accumulation is used at only a limited number of times throughout a movement, rather than continuously affecting the movement trajectories.

An important difference between the button-press and arm-reaching paradigms can be seen in the sensitivity of response latencies to the experimental manipulation of coherence. In the case of button-press latencies, RTs decreased with increasing coherence levels. In the case of reaching trajectories, movement initiation latencies did not vary with coherence levels. This illustrates the success of our experimental procedure, and demonstrates that subjects did not simply treat the reaching task in the same way as a button press task (but with the buttons far away). Rather, the data support our assumption that movement onsets are determined through a separate process, as assumed in our model.

In the models, we assumed that the onset time of the second submovement occurs approximately half way through the execution of the first submovement, and is not a function only of the evidence accumulated. This assumption was based on the correlation previously observed between the duration of the first submovement, and the onset time of the second submovement relative to the onset of the first submovement (Lee et al., 1997). Our assumption meant that in our models, the decision process played no part in the onset time of the second submovement. The validity of this assumption could be tested, for example, by modifying in some way the cost of moving in the wrong direction, which would encourage earlier use of the available information, which would allow the validity of this relationship to be tested.

There is an important distinction between our work and the work recently reported by Resulaj, Kiani, Wolpert, and Shadlen (2009). In their work, they also showed how an accumulator model could be used to describe arm movements in a perceptual decision making task. However, the focus of their study was on how information in the "processing pipeline" can lead to "changes of mind". More specifically, in their model, a decision is made when accumulation reaches a bound but, importantly, they allowed for the evidence accumulation process to continue after this initial decision. Interestingly, they were able to show that allowing the diffusion process to continue would occasionally lead to the opposite bound (decision) being reached. Critically, this "two-step" model was best able to describe the full pattern of reaching trajectories, which revealed that subjects had "changed their mind" on occasion. The focus of the study reported here is very different.

In contrast to Resulaj et al. (2009), our work has focused on movements initiated *before* a decision bound has been reached. In the two models implemented, a two-submovement movement is made (i.e., not straight to the target) when a final decision has not been made before a movement initiation deadline. As others have observed in similar debates, it is not easy to adjudicate between models that allow the decision maker access to partially-completed processing, and models that do not. However, our analysis of the reaching movements in terms of submovements helped advance this debate. When motion begins before decision processing is finished, there are multiple submovements, and the first submovement begins very early, and so very little evidence accumulation has taken place. The model that does not allow partial information predicts that the movement should be unaffected by the stimulus, where as the model that allows partial information predicts the opposite. We observed a systematic difference between left- vs. right-favouring targets. This finding provides support that partial evidence accumulated at movement onset can be incorporated into the motor plan, and not only a binary decision to one target or the other.

A central upshot of the present study is that an intermittent model which assumes discrete submovements, some of which are incomplete by virtue of being initiated prior to the evidence accumulation process having reached a bound, is sufficient to describe the effects of stimulus quality and stimulus viewing time on reaching trajectories. Our assumption that prematurely elicited reaching movements are comprised of an initial, incomplete submovement is consistent with recent findings that reflex gains in the arm resulting from perturbations (Selen, Shadlen, & Wolpert, 2012) and corticospinal excitability (Klein-Flügge & Bestmann, 2012) both track the evolution of a perceptual decision. We suggest that it was our use of a movement initiation deadline that led subjects to initiate their movements prematurely.

We mentioned in the Introduction how our approach here has been motivated in part by the suggestion of Van der Wel et al. (2009), who proposed in their response to Spivey et al. (2005) that the cognitive "trajectory" through decision space influences the arm reaching movement at discrete time points. But it is important to highlight how our model differs from their model too.

Although both are based on the superposition of minimum-jerk submovements, the model suggested by van der Wel et al. was a serial model in the sense that a perceptual decision is made first followed by the generation of a movement. A central assumption of their work was that subjects produce an initial, pre-decision submovement directly between the targets, followed by a second submovement after a decision has been made. The superposition of these submovements generated curved movements, similar to those observed in Spivey et al. (2005). Thus, their model assumed that the "pre-decision" movement was uninformed and, hence, straight between the two possible targets. In sharp contrast, our work here has shown that subjects' premature or "pre-decision" movements are sensitive to the current state of the accumulation process. Thus, while both our model and that of van der Wel et al. posit intermittent control, we have reached a very different conclusion from that of Van der Wel et al. (2009). Namely, we suggest that the motor system is informed by the available amount of accumulated evidence at discrete points throughout the movement. This limited use of the accumulated evidence is sufficient to describe the observed trajectories.

While previous works have shown that arm movements are exquisitely sensitive to decision making processes (Awasthi, Friedman, & Williams, 2011; Finkbeiner, Song, Nakayama, & Caramazza, 2008; Freeman & Ambady, 2009; Song & Nakayama, 2008b, 2009; Spivey et al., 2005), our study is the first one to establish that these movements can be described by a model that assumes movements are comprised of discrete elements (submovements) under intermittent control. Similar to Van der Wel et al. (2009), we offer our work here as an existence proof that curved reaching movements can be accounted for by a model which assumes that overt arm movements are under intermittent cognitive control.

4.1. Conclusions

We demonstrated that arm movements can be used to model decision-making processes in a similar way to using reaction times. Our models are based on the notion that while evidence accumulation is continuous, the motor system only accesses this information at a number of discrete times during a movement. The success of our models suggests that it is sufficient to posit intermittent control of arm movements to describe the systematic effects of stimulus quality and viewing time in curved arm movements. Our approach also suggests that movements which are evoked before a perceptual decision has been reached are nevertheless sensitive to the *partially*accumulated evidence.

Acknowledgments

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Appendix. Parameters for predicting arm movements

The following describes the parameters that were fit to the data in order to generate the predicted arm movements. All parameters were fit to the trajectories for each subject, combined from all conditions.

The duration of one submovement trials, D_1, was sampled from a normal distribution fit to the data, with average values of $\mu = 0.80$ s, $\sigma = 0.10$ s.

For two submovement trials, the amplitude of the first submovement A_1 was sampled from a normal distribution fit to the data, with average values of $\mu = 0.28$ m, $\sigma = 0.09$ m. The duration of the first submovement was, for each trial, sampled from a normal distribution with on average a mean of $D_1 = 0.63\,A_1 + 0.45$ s, and a standard deviation of 0.11 s. The second submovement was assumed to start a proportion of the way through the first submovement. On average this value was $t_1 = 0.77\,D_1 + t_0 - 0.20$ s. The duration of the second submovement was, for each trial, sampled from a normal distribution with on average a mean of $D_2 = 0.93\,A_2 + 0.26$ s, and a standard deviation of 0.09 s.

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