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Moving objects by imagination? Amount of finger movement and pendulum length determine success in the Chevreul pendulum illusion

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ABSTRACT

Hand-held pendulums can seemingly oscillate on their own, without perceived conscious control. This illusion, named after Chevreul, is likely a result of ideomotor movements. While this phenomenon was originally assumed to have a supernatural basis, it has been accepted for over 150 years that the movements are self-generated. However, until now, recordings of the small movements that create these oscillations have not been performed. In this study, we examined how participants produce these unconscious oscillations using a motion capture system. As expected, the Chevreul pendulum illusion was produced when the fingers holding the pendulum generated an oscillating frequency close to the resonant frequency of the pendulum, where very small driving movements of the arm are sufficient to produce relatively large pendulum motion. We found that pendulum length significantly affected the ability to produce the illusion - participants were much more successful with a 40 cm compared to an 80 cm pendulum. Further, we found that participants that tended to move their fingers more were more successful in producing the illusion but did not find a connection between inter-joint coordination and ability to generate the illusion.

1. Introduction

Can humans cause movements in external objects without perceiving that they are generating the movement? Are there motor actions that result from thoughts or mental images and are potentially instantiated independently of conscious engagement? Once attributed to external spirits, non-conscious motions of the hand-held pendulum have been attributed to ideomotor phenomenon. Ideomotor theory posits that actions are represented by their perceivable effects (Shin, Proctor, & Capaldi, 2010). This theory has been used as a way of explaining how voluntary movements can occur when one is not consciously aware of making movements. A classic example of this is when a hand-held pendulum will start moving without the holder feeling like they are performing any movement. Is it possible that participants can make movements and yet not realize that they are making them?

In 1808, Antoine-Claude Gerboin from Strasbourg School of Medicine described his observation of how the hand-held pendulum

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would move mysteriously when the person held it over certain substances. In the 1830s, a French scientist, Michel Eugene Chevreul studied the movement of such a hand-held pendulum in several situations (Chevreul, 1854; Jastrow, 1962), verifying that the movement decreased when the arm was being supported at the hand in contrast to the arm being externally supported at the shoulder, and that the oscillations were sight dependent. Chevreul postulated that imperceptible muscle activations were responsible for the pendulum's first oscillations, which increased (but were still imperceptible) under the influence of visual feedback (Chevreul, 1854). Although nearly 200 years have passed since this and similar examples of ideomotor behavior, such as Ouija board spelling were explained (Spitz, 1997), people are still being deceived by modern versions of ideomotor behavior, including tragically in facilitated communication (Burgess et al., 1998).

There have been several attempts to better understand the Chevreul pendulum illusion. In the 1970s, Easton and Shor used photogrammetry (Easton & Shor, 1975; Easton & Shor, 1976) and confirmed that sight is an important factor in determining when the pendulum will oscillate (Easton & Shor, 1975), and that as more attention is directed toward the pendulum, the more it oscillates (Easton & Shor, 1976). They also confirmed that restraining the arm at the wrist decreased the pendulum oscillation and demonstrated that out of 75 participants, only 60 were able to create the illusion (Easton & Shor, 1976). Hypnosis research uses Chevreul's pendulum illusion as a tool for testing a patient's response to the technique, with patients that are not able to move the pendulum generally being unresponsive to hypnosis (Karlin, Hill, & Messer, 2007). The studies of Easton and Shor, however, were not sensitive enough to describe how the participants generate the pendulum motion. Despite the early research and ongoing public fascination with these phenomena, there is a relative paucity of research examining the mechanistic accounts of 'automatic' pendulum oscillations.

A pendulum has a resonant frequency that is primarily dependent on its length. The maximum oscillation amplitude for a hand-held pendulum will be achieved if the driving frequency (i.e. the frequency of oscillations of the hand) is equal to the resonant frequency (Newburgh, 2004). Thus, in order to make the pendulum oscillate significantly, the participants are required to oscillate the pendulum-holding fingers at a frequency close to the resonant frequency of the pendulum. In this exploratory study, we set out to examine the relationship between generated arm movement and success in producing pendulum motion. We purposely chose pendulums that had natural frequencies of approximately 0.7, 1.05 and 2.0 Hz, in order that higher-frequency physiological tremor, e.g. 3–5 Hz for the elbow and 8–12 Hz for the finger (Hallett, 1998), would not produce significant pendulum motion. In this way, any significant pendulum motion must be a result of the participant moving their arm at an appropriate frequency. Using motion capture equipment, we aim to investigate how the total amount of movement of the fingers, and the coordination between joints of the arm affect successful performance of the illusion. Further, we wish to examine at which frequencies participants are able to generate pendulum oscillations, and how the different joints contribute to the movement of the pendulum.

2. Methods

2.1. Participants

Thirteen right-hand dominant participants (9 females), with normal or corrected vision took part in the study. The sample size was

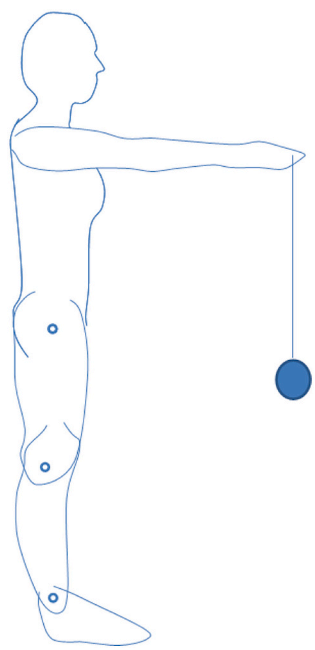

Fig. 1. A sketch of how the participants held the pendulum.

based on a pilot experiment (Cantergi & Friedman, 2018), where significant effects were observed in a sample size of 10 participants. The experimental protocol received ethical approval from the Tel Aviv University Institutional Research Board, and participants signed an informed consent form before starting the experiment. All methods were carried out in accordance with relevant guidelines and regulations. Participants received payment of 40 shekels (approximately US$11) for participating in the experiment.

2.2. Procedure

For each trial, participants stood at a marked location, with their hand outstretched in front of them (see Fig. 1). A magnetic motion capture system (Polhemus Liberty), sampling at 240 Hz with 8 sensors was used to record the movement of the pendulum and the right upper limb, using the Repeated Measures software (Friedman, 2014). One sensor was used as the pendulum, the other sensors were placed on the thumb, index and middle fingernails, on the back of the palm, on the forearm, on the upper arm, and on the acromion. Each recording was for 120 s. A short period of rest between attempts was provided. Before starting the recordings, participants were instructed to perform upper arm movements (each lasting 5 s) of one of the seven degree of freedom in each case, to allow calculation of the joint centers and joint angles.

The participants first held their arm outstretched without the pendulum. In the second trial, they held the wire 40 cm from the sensor, with no attempt made to move it. In the third, fourth and fifth trials, they held the pendulum and were asked to attempt to cause it to move (without moving their hand). This was performed with a 40 cm pendulum (as before), then with a pendulum half the length (20 cm), and double the length (80 cm). Finally, in the sixth trial, they repeated the attempt to move the 40 cm pendulum, but without visual feedback (no vision condition).

In order to determine the resonant frequency of the pendulum, the wire was attached to a stand, pulled to 45 degrees, and left to swing on its own. The resonant frequency was then calculated from the reciprocal of the mean distance between the extreme values on one side. The resonant frequencies for the 20 cm, 40 cm and 80 cm pendulums were found to be 2.00 Hz, 1.05 Hz and 0.70 Hz respectively.

2.3. Data analysis

The position and orientation data from the 8 sensors were smoothed using a 4th order, two-way low-pass Butterworth filter, with a cutoff of 5 Hz. Joint centers and subsequently the joint angles were calculated using a previously described technique (Biryukova, Roby-Brami, Frolov, & Mokhtari, 2000). A fast Fourier transform (FFT) was performed on each trial on the thumb velocities. As the pendulum is likely to move close to its resonant frequency due to any finger movements, we classified successful trials as those where both the pendulum moved a considerable amount (peak-to-peak amplitude greater than 10°) and the thumb FFT showed its largest peak close (defined as less than 15%) to the resonant frequency of the pendulum. We compared the success rates between the 40 cm pendulum and the shorter (20 cm) and longer (80 cm) pendulums using a chi-squared test.

For each trial, we calculated two types of measures of performance to see whether they predicted successful movement of the pendulum. First, we calculated the arc length, defined as the total amount of movement of the thumb, relative to the shoulder, i.e., we calculated the unsigned distance between each two time points, and then summed them. Second, we performed single-trial uncontrolled manifold (UCM) analysis (Scholz, Schöner, & Latash, 2000; Shaklai, Mimouni-Bloch, Levin, & Friedman, 2017) in order to determine how the joints co-varied in their movements. The variance observed was decomposed into two components: "good" variance (variance within the uncontrolled manifold) that does not affect the task outcome (i.e., the location of the pendulum held in the hand), and "bad" variance (variance orthogonal to the uncontrolled manifold, which does affect the task outcome). In addition, we calculated the synergy index (ratio of good to bad variance), to quantify overall how well participants stabilized the location of the fingers. Full details of the analysis are presented in Appendix 1.

We used one-sided Wilcoxon signed rank tests to test the following hypotheses related to pendulum movement: 1) Arc length will be longer for successful compared to unsuccessful participants, because more finger movement can lead to more pendulum movement. 2) "Bad" variance will be greater in successful participants, as poorer control of the end-effector can lead to more movement of the pendulum. 3) "Good" variance and 4) the synergy index will be lower in successful participants (as they better stabilize the end-effector), so are likely to move the pendulum less.

We used one-sided Wilcoxon tests due to the non-normal distribution and the relatively small sample size so as to reduce the effect of outliers. We presented the W statistic for the test, which is the sum of the ranks of positive differences, along with the approximate 95% confidence intervals (Altman, Machin, Bryant, & Gardner, 2000). Statistical analyses were performed using Matlab 2018b (Mathworks).

In addition, we also determined the relative contributions of the body (not including the arm), the shoulder, elbow and wrist to left-right movements of the thumb in the relevant frequency range (within 0.2 Hz of the pendulum frequency). After calculating the joint angles of the body (relative to the fixed coordinate system of the motion capture system), the shoulder, elbow and wrist, we considered only the component responsible for rotations about the vertical axis (i.e., those that cause left-right movement). We then computed the joint velocity (by taking the derivative) and computed the tangential velocity at the thumb (by multiplying the joint velocity by the joint-thumb distance). We computed the relative contribution by dividing the area under the relevant region of the FFT for one joint by the sum of the areas for the four joints. Descriptive statistics (medians and 95% confidence intervals) were presented for this data.

3. Results

Some of the participants were successful in achieving the pendulum illusion. The success was greater when using the 40 cm pendulum with vision (62%). Lower success rates were achieved with the longer pendulum (46%), although the difference was not statistically significant ($\chi^2(1) = 0.286$, $p = 0.59$). A significantly lower success rate was shown with shorter pendulums (8%; ($\chi^2(1) = 5.444$, $p = 0.02$)). When no visual feedback was allowed (no vision condition), the success rate was lower (31%), but not significantly so ($\chi^2(1) = 1.333$, $p = 0.25$). All chi-squared test comparisons were to the 40 cm pendulum. Four participants (31%) were unsuccessful in all tasks. An example of a successful and unsuccessful participant for the 40 cm pendulum task with vision is shown in Fig. 2.

3.1. Arc length

We computed the arc length of the thumb in the different conditions, shown in Fig. 3(a). When attempting to move the pendulum at 40 cm, those successful in moving the pendulum showed greater arc lengths (median 240 cm, CI = [140, 881] cm) than those who were unsuccessful for that condition (median 169 cm, CI = [122, 180] cm; Wilcoxon rank sum test: W = 19, $p = 0.001$), as were those at 80 cm (successful: median 273 cm, CI = [152, 779] cm; unsuccessful: median 159 cm, CI = [89, 283] cm; W = 34, $p = 0.017$) and in the condition with no vision (successful: median 253 cm, CI = [203,301] cm); unsuccessful: median 177 cm, CI = [171, 224] cm; W = 50, $p = 0.025$.

3.2. UCM analysis

The UCM analysis did not reveal any significant differences between successful and unsuccessful individuals in this task. The results are summarized in Table 1, and the values of the synergy index for all participants are illustrated in Fig. 3b).

3.3. Relative contribution of the joints

We also examined the relative contributions of different joints to moving the pendulum, i.e. the effect of the body (not including the arm), shoulder, elbow and wrist velocities on left-right thumb velocity in the relevant (± 0.2 Hz from the pendulum frequency) for successful trials. The contributions of the joints are shown in Fig. 4. The body (not including the arm) contributed a median of 41.2% (CI = [34.6, 47.7]%) of the pendulum movement, which was the most movement for an overwhelming majority of the trials (15 out of 19). The shoulder contributed a median of 29.3% (CI = [21.4,33.7]%) to the pendulum movement, which was the most movement for

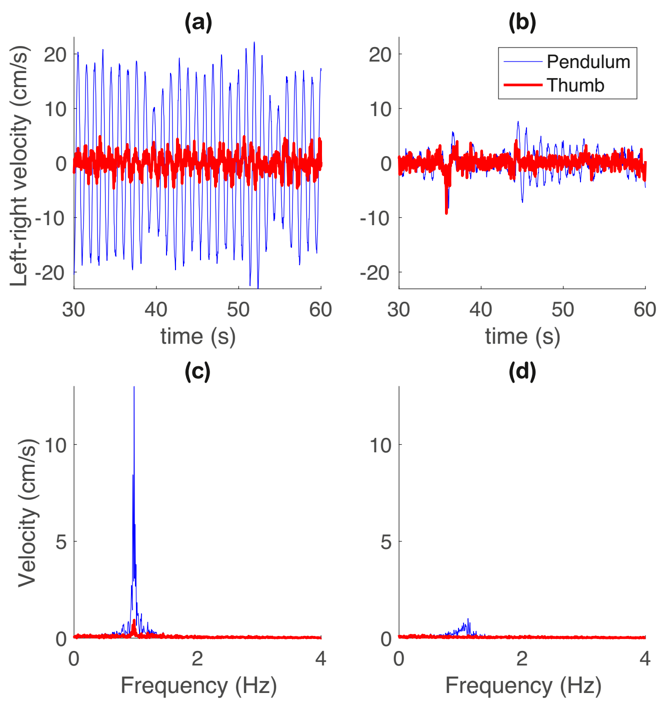

Fig. 2. Example of a successful (left column) and unsuccessful (right column) participant. (a) and (b) show the left-right velocity of the pendulum and the thumb (for a 30 s portion of the trial). (c) and (d) show the Fourier transforms (using data from the whole 2 min) – for the successful participant (c), a clear aligned peak can be observed at approximately the resonant frequency of the pendulum for both the pendulum and the thumb, whereas no peak is present for the thumb or pendulum at this frequency for the unsuccessful participant (d).

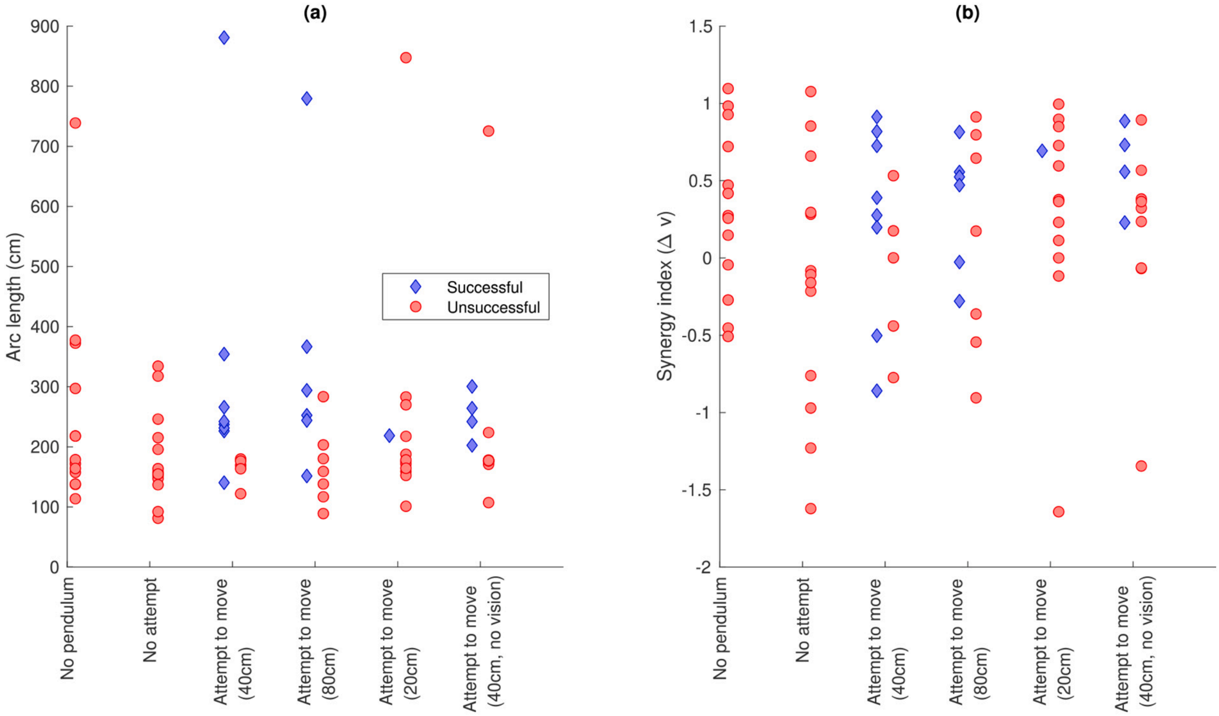

Fig. 3. a) Comparison of arc lengths (over two minutes) in the six different conditions. b) The synergy index Δv. In both graphs, blue diamonds indicate successful trials, red dots unsuccessful trials. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 1
Results of UCM analysis.

Task	Median successful	Successful CI	Median unsuccessful	Unsuccessful CI	W	P
$V_{bad} \times 10^{-4}$ (rad^2)						
Attempt to move (40 cm)	0.56	0.04–2.26	0.33	0.13–2.17	27	0.89
Attempt to move (80 cm)	1.94	0.31–5.47	0.88	0.17–5.97	43	0.82
Attempt to move (20 cm)	0.31	0.31–0.31	0.63	0.25–3.63	87	0.31
Attempt to move (40 cm, no vision)	1.10	0.33–1.73	0.40	0.27–2.89	60	0.70
$V_{good} \times 10^{-4}$ (rad^2)						
Attempt to move (40 cm)	0.90	0.07–7.12	0.16	0.08–2.57	28	0.86
Attempt to move (80 cm)	1.99	0.43–18.20	1.12	0.11–66.02	43	0.82
Attempt to move (20 cm)	0.50	0.50–0.50	1.67	0.36–4.21	87	0.31
Attempt to move (40 cm, no vision)	2.57	0.69–3.96	0.73	0.22–2.32	55	0.90
Δv						
Attempt to move (40 cm)	0.22	−0.71 - 1.04	0.14	−0.86 - 0.39	29	0.82
Attempt to move (80 cm)	0.53	−0.43 - 0.85	−0.07	−0.62 - 1.18	41	0.88
Attempt to move (20 cm)	0.41	0.41–0.41	0.43	0.02–0.76	84	0.54
Attempt to move (40 cm, no vision)	0.61	0.40–0.79	0.16	−0.23 - 0.61	51	0.97

The medians and 95% confidence intervals are presented for participants who were successful and not successful for the tasks. The right two columns are the W statistic and *p*-value for one-sided Wilcoxon tests for the hypotheses: V_{bad} successful > V_{bad} unsuccessful, V_{good} successful < V_{good} unsuccessful, and Δv successful < Δv unsuccessful.

1 trial. The elbow contributed a median of 16.3% (CI = [11.8, 19.6]%), which was the most movement for 2 trials. The wrist contributed a median of 12.3% (CI = [9.3, 21.0]%), which the most movement for 1 trial.

4. Discussion

Hand-held pendulums can seemingly oscillate on their own, without perceived conscious control. This phenomenon, known as the Chevreul pendulum illusion, is likely a result of subtle muscle movements caused by thinking of the generated movement. In this study, we examined how a hand-held pendulum, at three different resonant frequencies, results in significant oscillations that drive the

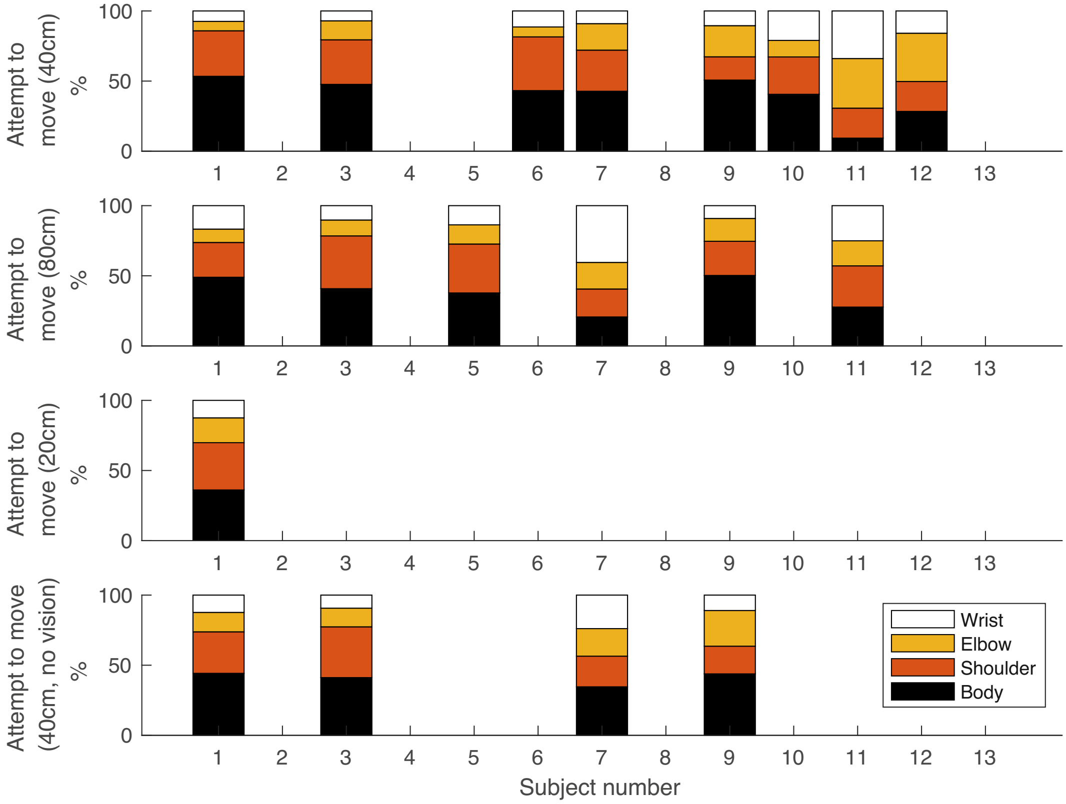

Fig. 4. Relative contributions (%) of the body (not including the arm), wrist, elbow and shoulder to left-right (y) movements of the pendulum. Only trials where participants successfully oscillated the pendulum are shown.

pendulum movement without conscious causal agency. We demonstrated that the movement of the pendulum is produced by oscillating the fingers holding the pendulum at a frequency close to the resonant frequency of the pendulum. At an appropriate frequency, very small driving movements of the body and arm were sufficient to produce relatively large pendulum motion, when the pendulum was sufficiently long (40 cm or 80 cm) but not for a 20 cm pendulum.

Participants that move their hands more were more successful in producing the illusion. Perhaps, in agreement with Chevreul's proposal, as they move more, the pendulum is more likely to start moving, and then they reinforce these movements. In a letter to Ampere, Chevreul argued that the tendency of movement in a specific direction is caused by the attention on the pendulum, that is reinforced by sight of the holding hand which extends the movement (oscillations) further. In many other cases, like bowling or billiards, while following a movement with our eyes, we tend to move our bodies in the direction we want the moving body to follow, as if directing the movement toward the goal (Spitz & Marcuard, 2001). The observation that participants with higher movement show greater likelihood of pendulum oscillation agrees with the bi-directionality relationship between thought and action (i.e. thought of movement resulting in actual movement for participants), consistent with their expectations (Shin et al., 2010). According to Koch, Keller & Prinz (Koch, Keller, & Prinz, 2004), the anticipation of response effects likely serves as a mental cue to activate corresponding movements.

The resonant frequency of these pendulums allowed the very small movements of the hand to be magnified in the movements of the pendulum, where they could be cumulatively built upon until a regular swinging motion ensued. Why were many participants successful at 40 cm and 80 cm and not 20 cm? The resonant frequency for 20 cm is relatively high (2 Hz) but still lower than tremor, and similar to the frequency of typical movements (whereas the others are relatively low). It is unclear why it seems difficult to unconsciously produce movements at this frequency. A possible explanation is that the 40 cm and 80 cm pendulums have resonant frequencies (1.05 Hz and 0.7 Hz respectively) that are close to the natural frequency of the arm, which is approximately 1 Hz (Wagenaar & van Emmerik, 2000), whereas the 20 cm pendulum has a resonant frequency (2 Hz) which is far from the resonant frequency of the arm. We note that participants were more successful with the 40 cm pendulum, which is closer to the resonant frequency of the arm.

We observed that most of the pendulum movement resulted from movements of the body and the shoulder. Part of this may be due to the longer lever arm of the body and shoulder – a small rotation of the shoulder causes a much larger rotation at the fingers compared to a rotation of the wrist. It may also be that as participants are not able to directly observe their body sway and shoulder rotations while performing the task (if they are looking at the pendulum), the sense of agency that occurs due to visual feedback of one's movement is lacking (Haggard, 2017).

Another type of movement where participants do not feel agency of the movement is the Kohnstamm phenomenon. This phenomenon occurs when the arm is pressed hard outwards for approximately 30 s, and then moved away from the surface. As a result of this, the arm involuntarily moves upward. A modeling approach to this phenomenon explains that the lack of agency over the movement is likely a result of the lack of an efference copy (De Havas, Ghosh, Gomi, & Haggard, 2015; De Havas, Gomi, & Haggard, 2017). In the Chevreul pendulum illusion, we speculate that the very small movements of the body and arm (in contrast to the relatively large pendulum movements) may result in a relatively small efference copy, which may reduce the sense of agency over the movements.

After adapting to a force field or visual rotation, participants will unknowingly produce movements biased in a particular direction after the perturbation has been removed (Scheidt, Reinkensmeyer, Conditt, Rymer, & Mussa-Ivaldi, 2000). Similarly, after walking on a rotating platform, participants will inadvertently turn in circles after the platform has stopped rotating, known as podokinetic after-rotation (Earhart & Hong, 2006). In both these cases, participants are aware that they are making a movement, but are often unaware of the direction of the movement generated by the after-effects (as visual feedback is not available). These examples show that even though proprioceptive feedback is available in both these examples, it is insufficient to change the consciously perceived direction of the movement. As our bodies are constantly moving (e.g., due to postural sway), we likely often ignore these small shifts in position. In this illusion, the movement of the pendulum (when successful) is much larger than the movements of the body and the arm. Thus, it is possible that participants incorrectly assign the consequences of their movement, assuming them to have no net effect on the hand holding the pendulum.

Some researchers (Karlin et al., 2007; Olson, Jeyanesan, & Raz, 2017) posit that ideomotor phenomena have large interpersonal variability, wherein the hand-held pendulum oscillations are caused due to different decision strategies. Karlin et al. (Karlin et al., 2007) showed lower hypnotic susceptibility in participants that could not produce the Chevreul's Pendulum illusion, while Olson et al. (Olson et al., 2017) showed one of the personality measures, transliminality (sensitivity to subtle stimuli) predicted pendulum performance.

Studying a group of Ouija enthusiasts in a field experiment, Anderson et al. (Andersen et al., 2019) reported that a combination of retrospective inference and an inhibition of predictive processes caused a reduced sense of agency (subjective experience of not moving the planchette themselves) in Ouija board believers. Herbort & Butz (Herbort & Butz, 2012) proposed a computational model of ideomotor theory arguing that action-effect associations are boosted when acting in an *intention-based* action mode. Our finding, that success with pendulum oscillations was twice as high compared to when no visual feedback was allowed, are in line with ideomotor theories that action intentions are essentially perceptual. Mentalizing the movement results in higher success rate when salient visual cues are available.

Rather than a response to a sensory stimulus, ideomotor actions are unconsciously initiated and the results here also demonstrate that priming or thinking of motion can induce muscle micromovements (not visible to the naked eye) that end up moving the pendulum. In light of contemporary ideomotor theory (Shin et al., 2010), knowledge of stimulus-response compatibility is a critical component in the pendulum illusion. An action is automatically associated with its effect and that anticipation of the effect facilitates action in a bidirectional manner. Ideomotor actions underlie similar phenomena where items reportedly move of their own accord, like dowsing, facilitated communication or automatic writing, as well as in planchette motions/Ouija phenomenon. In these cases, our intentional stance and sense of agency is likely diminished (Wegner, Fuller, & Sparrow, 2003).

This study has some potential limitations. As there are no previous studies using motion capture to examine this phenomenon, the analysis was largely exploratory. Due to this exploratory nature of the analysis, corrections were not performed for multiple comparisons. As a consequence, caution is appropriate in interpreting these results. The relatively small sample size also limits the conclusions that can be drawn from this study. The fixed order of the trials may have led to order effects, and may explain why the results without vision differed from that in previous studies.

Following on from a scientific tradition of explaining the mechanisms behind seemingly supernatural phenomena (Pfungst, 1911), in this study we demonstrated in detail for the first time using objective motion capture data that the movements of the pendulum result from small oscillations of the hand, and showed that oscillations close to the resonant frequency of the pendulum produce this movement. Further, in an exploratory analysis, we showed that participants who move their fingers more are more likely to produce the illusion, and that most of the movement comes from the trunk rather than the arm. This adds to the existing literature on possible mechanisms underlying movements that are not consciously caused but are seemingly automatic. This leads to many further questions related to the unconscious production of movement, for example, whether there is a limit on frequencies of movement that can be produced without conscious awareness, and whether the movements produced are a result of prior physics knowledge or simply a result of trial-and-error. Understanding these phenomena may help explain how other types of unconsciously generated movements are produced.

Data availability

The raw data and analysis software are available for download from figshare: doi: https://doi.org/10.6084/m9.figshare.14708883

Author contributions

JF and BA developed the study concept. Data collection were performed by DC. JF performed the data analysis. DC drafted the manuscript, and JF and BA provided critical revisions. All authors approved the final version of the manuscript for submission.

Competing interests

The authors declare no competing interests.

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Appendix 1. Uncontrolled manifold (UCM) analysis

As each trial was performed under different instructions, we analyzed each trial individually, which is usually described as single-trial UCM (Scholz, Kang, Patterson, & Latash, 2003; Shaklai et al., 2017). In order to do this, we decompose the variance into the component which affects the end-effector ("bad" variance), and the component which does not ("good" variance). We derived the formulation for the "good" component of the variance by finding the Jacobian (relationship between changes in joint angles and changes in end-effector position).

First, we wrote the equation describing the position of the end-effector (the fingers) as a function of four angles (θ_b, θ_s, θ_e, θ_w) – corresponding to the angles of the body, shoulder, elbow and wrist. Body angle is defined as the orientation of the sensor on the acromion relative to the coordinate system of the motion tracking system. Each angle only considered movements about a vertical axis (i.e., those which can cause left-right, or front-back movements of the end-effector). The left-right position *y* of the end-effector can then be written as:

$$y = sin(\theta_b)\, l_1 + sin(\theta_{b+}\, \theta_s)\, l_2 + sin(\theta_{b+}\, \theta_{s+}\, \theta_e)\, l_3 + sin(\theta_{b+}\, \theta_{s+}\, \theta_{e+}\, \theta_w)\, l_4$$

where l_1, l_2, l_3 and l_4 are the lengths from the acromion to the shoulder joint, from the shoulder joint to the elbow joint, from the elbow joint to the wrist joint, and from the wrist joint to the fingers, respectively. We only considered the left-right position because forward-back movements would not cause the pendulum to swing in the instructed direction. We then calculated the Jacobian (i.e., the matrix that describes the relationship between joint velocities, and left-right changes in position of end-effector):

$$J = \frac{\partial y}{\partial \theta} = \begin{bmatrix} l_1 cos(\theta_b) + l_2 cos(\theta_b + \theta_s) + l_3 cos(\theta_b + \theta_s + \theta_e) + l_4 cos(\theta_b + \theta_s + \theta_e + \theta_w) \\ l_2 cos(\theta_b + \theta_s) + l_3 cos(\theta_b + \theta_s + \theta_e) + l_4 cos(\theta_b + \theta_s + \theta_e + \theta_w) \\ l_3 cos(\theta_b + \theta_s + \theta_e) + l_4 cos(\theta_b + \theta_s + \theta_e + \theta_w) \\ l_4 cos(\theta_b + \theta_s + \theta_e + \theta_w) \end{bmatrix}$$

We then found the null-space *e* of this matrix, i.e. combinations of joint angles that do not affect the horizontal position of the end effector (in order to find the good variance):

$$0 = J\, e_i$$

where e_i is the *i*th column of the null-space matrix (there are three such columns).

After detrending the data (with a cubic polynomial) to remove slow shifts in posture such that the relative joint angles are centered around zero, we projected these values $d\theta$ onto the null-space vectors to find the uncontrolled manifold, which will be a 3D linear space in the 4D space of joint angles. We summed them to find the component parallel to the uncontrolled manifold $f_{||}$:

$$\theta_{\|} = \sum_{i=1}^{3} \left(e_i^T \bullet d\theta\right) e_i$$

This is the component that does not affect the performance variable (end-effector horizontal position), so the remainder of the variance must affect the performance variable, i.e.,

$$\theta_{\perp} = d\theta - \theta_{\|}$$

We now find the "good" part of the variance, normalized by the dimension of the UCM:

$$v_{good} = \sum_{i=1}^{N_{samples}} \frac{|\theta_{\|}|^2}{3N_{samples}}$$

Similarly, we can find the "bad" part of the variance (which does affect the end-effector position):

$$v_{bad} = \sum_{i=1}^{N_{samples}} \frac{|\theta_{\perp}|^2}{N_{samples}}$$

Finally, we calculate the synergy index, which is the difference between good and bad variance, normalized by the total variance in the joint angle space in which it was computed:

$$\Delta v = \frac{v_{good} - v_{bad}}{\left(3v_{good} + v_{bad}\right)/4}$$

The synergy index Δv can range from -4 (signifying all variance is bad variance) to $+4/3$ (signifying all variance is good variance).

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